Temporary modulation of the number sense changes children's symbolic math performance

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Abstract

Children are born with an Approximate Number System (ANS) that supports an intuitive sense of number. Although previous work suggests that individual differences in ANS precision correlate with symbolic mathematics abilities in both children and adults, this work is almost entirely correlational in nature. Here, we offer direct evidence for a causal link between ANS precision and symbolic math performance in preschoolers by demonstrating that temporary modulations of ANS precision uniquely modulate subsequent symbolic math performance. In four experiments, we extend a recent finding that ANS precision can be temporarily modulated by trial order (i.e., an ANS confidence hysteresis effect; Odic, Hock & Halberda, 2014): children showed better ANS precision when number discrimination trials progressed from easiest to hardest (Easy-First ANS training condition) and showed worse ANS precision when the very same trials progressed from hardest to easiest (Hard-First ANS training condition). Critically, this brief modulation of ANS precision transferred to performance in a subsequent symbolic math task, with children in the Easy-First ANS training condition outperforming children in the Hard-First ANS training condition on subsequent symbolic math as well. This link was further unique to symbolic math: we found no transfer of ANS training to tests of verbal ability. Finally, we found a developmental effect of this modulation: while 5-year-olds showed transfer across all symbolic math categories tested, 4-year-olds were selectively impaired on a subset of symbolic math skills. These results demonstrate the malleability of ANS precision through a brief, 5-minute ANS training task and demonstrate a specific causal link from the ANS to symbolic mathematics.

Keywords: Approximate Number System (ANS); causal relationship; symbolic math; confidence hysteresis

Increasing a student's mathematical competence is a central goal of formal education. Yet skill at solving math problems and comprehending math concepts varies greatly across individuals. Whereas some children show great difficulty in mastering math facts, with these difficulties persisting over many years (Geary, 2004), other children exhibit outstanding abilities in advanced mathematics prior to adolescence (Brody & Mills, 2005). Given the importance of formal mathematics ability for job attainment, salary, and personal debt (Rivera-Batiz, 1992; Dougherty, 2003; Parsons & Bynner, 2005; Gerardi, Goette, & Meier, 2013), there is a pressing need to understand the sources of individual variability in mathematics comprehension and performance.

Recent work in cognitive development suggests that long before formal education has begun, children have an intuitive, non-symbolic, *approximate* sense of number, supported by an Approximate Number System (ANS; for review see Feigenson, Dehaene & Spelke, 2004; Nieder & Dehaene, 2009; Dehaene & Brannon, 2011; Halberda & Odic, in press). The ANS is functional even in newborn infants (Izard, Sann, Spelke, & Streri, 2009), is used by adults lacking formal math education (Pica, Lemer, Izard, & Dehaene, 2004; Frank, Everett, Fedorenko, & Gibson, 2008), has been found in a wide range of non-human species (Dehaene, Dehaene-Lambertz & Cohen, 1998; Cantlon, Platt & Brannon, 2009), and responds to numerosity across several sensory modalities (Barth, Kanwisher, & Spelke, 2003; Izard et al., 2009; Feigenson, 2011; Nieder, 2012; Libertus, Feigenson, & Halberda, 2013a). The ANS represents numerical quantity in a noisy, imprecise fashion, with the imprecision of its numerical representations growing with the target numerosity. Hence, the ability to nonverbally numerically discriminate two arrays depends on the ratio of the arrays rather than their absolute difference (Whalen, Gelman, & Gallistel, 1999; Piazza, Izard, Pinel, Le Bihan & Dehaene, 2004; Halberda & Odic, in press). For example, discriminating 8 vs. 16 dots (a ratio of 2.0) is as easy as discriminating 20 vs. 40 dots, and is easier than discriminating 32 vs. 40 dots (a ratio of 1.25).

Whereas some individuals can discriminate arrays that are very close in number (i.e., ratios near 1), others struggle to discriminate arrays that differ by even very large ratios (Halberda, Mazzocco, & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman &

Germine, 2012). These individual differences in ANS precision can be seen prior to mathematics education, in both preschool-aged children (Libertus, Feigenson, & Halberda, 2011; 2013b) and in pre-verbal infants (Libertus & Brannon, 2010; Starr, Libertus & Brannon, 2013). Additionally, ANS representations grow in precision over development from infancy through early adulthood (Lipton & Spelke, 2003; Halberda & Feigenson, 2008; Piazza, et al., 2010; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Odic, Libertus, Feigenson & Halberda, 2013), and then appear to slowly decline in precision later in life (Halberda et al., 2012).

What role might ANS representations play in the symbolic mathematics that children encounter in school? On the surface, ANS and symbolic math appear to have little in common. Whereas the ANS operates over imprecise, inherently noisy representations, symbolic mathematics mostly requires precise, discrete representations of numbers. Whereas the ANS is present without instruction or language (Izard et al., 2009), symbolic mathematics requires formal instruction and is often notoriously difficult for children to master (e.g., Berch & Mazzocco, 2007). And whereas the ANS is shared with non-human animals such as rats, pigeons, and guppies (Meck & Church, 1983; Roberts, 1995; Piffer, Agrillo & Hyde, 2012), symbolic math appears to be a uniquely human ability (Dehaene, 2003; Cantlon, 2009).

Yet despite these differences between the ANS and formal mathematics, an emerging body of research suggests that approximate number representations play an important role in symbolic math performance, as individual differences in ANS precision are linked to formal mathematics achievement in both children and adults. ANS precision correlates with performance on standardized math tests that were taken several years previously (Halberda et al., 2008; Libertus, Odic, & Halberda, 2012), correlates with math performance on concurrent assessments (Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus et al., 2011; Lyons & Beilock, 2011; Lourenco et al., 2012), and predicts future math performance (Gilmore, McCarthy, & Spelke, 2010; Mazzocco, Feigenson, & Halberda, 2011a; Libertus et al., 2013b; Starr, Libertus, & Brannon, 2013). Children with mathematical learning disabilities (MLD, or dyscalculia) have significantly poorer ANS precision than typically developing children (Piazza et al., 2010; Mazzocco, Feigenson & Halberda, 2011b). Although still controversial (Iuculano, Tang, Hall & Butterworth, 2008;

Holloway & Ansari, 2009; Soltesz, Szucs, & Szucs, 2010; De Smedt, Noel, Gilmore, & Ansari, 2013) and in need of continued investigation, the sum total of current findings suggest that the link between ANS representations and symbolic mathematical competence may start early in life and persist throughout development (Feigenson, Libertus, & Halberda, 2013; Chen & Li, 2014).

The evidence demonstrated so far for a link between the ANS and math abilities has been almost entirely correlational (e.g., individual differences in ANS precision correlate with symbolic math performance). Although many of the above studies controlled for non-numerical cognitive and motivational factors including general IQ and working memory, a direct causal link between the ANS and symbolic math abilities has yet to be demonstrated in preschool children. Ideally, to demonstrate such a link, researchers would experimentally manipulate the precision of children' ANS representations, and then measure the effect this has on their symbolic math abilities.

Evidence for the malleability of the ANS and its effect on subsequent symbolic math performance remains sparse. Wilson and colleagues (2006) showed that after 5 weeks of playing an adaptive computer–assisted intervention, the Number Race Game, 7- to 9-year-old children with math learning difficulties showed improved performance in an ANS discrimination task. Räsänen and colleagues (2009) subsequently showed that 6-year-old children trained this way performed better in a symbolic number comparison task compared to pretests (e.g., judging which symbolic number is larger in magnitude), but they showed no improvements in other mathematical abilities, such as counting or arithmetic tasks. Although this effect may be interpreted as support for malleability of ANS in low-achieving school-age children, it could also be accounted for by a simple practice effect of magnitude comparison (Räsänen et al., 2009), which is known to be trainable (DeWind & Brannon, 2012).

Recently, two studies have shown that training in ANS-based arithmetic tasks has benefits for subsequent symbolic arithmetic performance. Park and Brannon (2013, 2014) had adults complete about 10 days of training in which they saw non-symbolic dot arrays being added or subtracted, then had to compare the approximate answer to a third array. After ten training sessions (requiring a total of approximately four hours), accuracy in the approximate arithmetic task had significantly improved. Critically, this improvement in approximate arithmetic transferred to performance in a subsequent symbolic addition and subtraction task (i.e., the total number of problems participants solved correctly in 10 minutes improved by 0.4 standard deviations of their pre-test performance), but not to performance in a verbal task. Hyde and colleagues (2014) extended this arithmetic training result to first-grade children. In their study, children were randomly assigned to one of four 10-minute training sessions: approximate numerical addition, approximate numerical comparison, line length addition, and brightness comparison. Despite the fact that children's ANS precision was not changed by the training, children who practiced in either an approximate numerical addition task or an approximate numerical comparison task were significantly faster at solving subsequent symbolic arithmetic problems compared to children who practiced non-numerical tasks such as line length addition or brightness comparison (Hyde, Khanum, & Spelke, 2014).

Although the evidence from both Park and Brannon (2013) and Hyde and colleagues (2014) is in support of a causal link between ANS representations and symbolic mathematics, it leaves two important questions unanswered. First, both Park and Brannon (2013) and Hyde and colleagues (2014) tested participants who already had symbolic math education and experience with arithmetic computations. If the ANS plays a foundational role in children's symbolic math abilities, we should find that enhancing ANS precision affects symbolic math performance even in children with little or no formal math education. Second, the evidence for the causal link between the ANS and symbolic math so far is restricted to efficiency at solving addition and subtraction problems or magnitude comparisons in symbolic and ANS contexts, rather than showing that improvements in ANS *precision* affects accuracy in symbolic math tasks. As a result, it remains unknown whether the observed improvements in symbolic arithmetic are due to exercising the ANS, or due to practicing domain-general arithmetic computations (as suggested by Park & Brannon, 2014, practicing computations over ANS representations, rather than exercising the ANS itself, was the key to adults' improvement on symbolic arithmetic performance). If ANS is directly linked to symbolic math abilities, we should find that enhancing ANS precision alone results in changes in various aspects of symbolic mathematics that have been found to correlate with ANS precision, such as reading and comparing number symbols (Libertus et al., 2013a).

In the present series of experiments, we asked whether modulating ANS precision affects a broad set of symbolic math abilities in children with little or no formal math education. Our experiments build upon a recent method that allows for quick and temporary modulation of ANS precision in young children, termed *ANS confidence hysteresis* (Odic, Hock, & Halberda, 2014). In previous work, Odic and colleagues presented 4- to 6- year old children with a non-symbolic ANS discrimination task. Critically, they manipulated the order in which children were presented with easy versus difficult approximate numerical discriminations. When children completed a sequence of Easy-First trials that began with easy discriminations (e.g., ratios such as 3.0, and 2.0) and gradually moved to more difficult discriminations (e.g., ratios such as 1.10, and 1.07), children exhibited significantly better ANS precision compared to those who completed the identical trials in a Hard-First order that progressed from hard to easy ratios. Additional control conditions showed that this performance difference was not due to general factors such as loss of interest, amount of practice, or poor self-evaluation in the Hard-First condition (see Odic et al., 2014 for details). Instead, children appear to have experienced a temporary change in their ANS precision (or in their ability to appropriately use ANS representations) – an effect the authors termed ANS *confidence hysteresis* to highlight connections to literatures in dynamical psychophysics (Hock & Schoner, 2010).

Here we harnessed the confidence hysteresis effect in order to study the relationship between ANS representations and symbolic math performance in preschool children. In Experiment 1a we presented 5-year-old children with numerical discriminations that progressed either in the Easy-First, Hard-First, or Random order, then tested children's performance on a symbolic math transfer task. If the ANS plays a foundational role in symbolic mathematics prior to formal education, then symbolic math performance should be enhanced in the Easy-First training group relative to the Hard-First training group, with the Random control group in-between. Then, in Experiment 1b, we tested the specificity of ANS confidence hysteresis and the possibility that the transfer effects might reflect changes in general motivation or self-efficacy. A separate group of children completed either the Easy-First order or the Hard-First order of the ANS training task, and then were tested on a vocabulary transfer task (cf. Park & Brannon, 2013; Hyde et al, 2014). If the ANS plays a specific role in symbolic mathematics prior to formal education, then vocabulary performance should not be affected by ANS training. Finally, in Experiments 2a and 2b, we extend these findings to even younger children and investigate whether the transfer effect is specific to only a subset of symbolic math abilities.

Experiment 1a: ANS Confidence Hysteresis and Symbolic Math in 5-Year-Olds

In order to test whether temporary changes in ANS acuity affect symbolic math performance, we first induced confidence hysteresis in ANS representations by replicating the finding that ANS precision is affected by the order of trial difficulty (Odic et al., 2014). We randomly assigned one third of the children to the Easy-First ANS training condition, one third to the Hard-First ANS training condition, and one third to the Random control condition. In the Easy-First training condition, the experiment began by presenting children with trials containing easily discriminable numerical ratios and gradually progressed through trials containing harder numerical ratios. In the Hard-First training condition, children experienced the mirror-reverse: they started with the most difficult trials and gradually progressed through trials that decreased in difficulty. In the Random control condition, trials were predetermined in a pseudo-random order, intermixing the easier trials with harder trials (see Appendix A for a list of the ratio sequences).

Next, to ask whether any observed effects of ANS confidence hysteresis would transfer to children's symbolic math performance, we immediately followed the ANS Training task with a Symbolic Math Transfer task, which was comprised of a broad subset of items from the TEMA-3, including scales on numbering, number comparison, calculation, and numerical literacy (Test of Early Mathematics Ability, Ginsburg & Baroody, 2003). These scales are known to correlate with ANS precision (Libertus et al., 2013a) and further develop at different times in 5-year-olds (Ginsburg & Baroody, 2003).

Method

Participants. Thirty children with a mean age of 5 years, 3 months (*SD* = 2.14 months; range = 60.30 months to 67.30 months; 16 females) participated, with 10 children assigned to each Condition of the ANS Training task (Easy-First, Hard-First, or

Random). Two additional children (one from each condition) were excluded from the final sample because they were unable to complete the Symbolic Math Transfer task – failing to correctly answer more than one of the first three questions (i.e., the easiest three questions). Because we did not have any practice items in our transfer tasks, this criterion was used throughout all experiments as a check that children understood the task. Children received a small gift (e.g., T-shirt, book, or toy) to thank them for their participation.

ANS Training task. The ANS Training task was designed to measure children's ability to perform nonverbal numerical discriminations and to manipulate children's ANS precision through a very brief, 5-minute intervention. This task was a direct replication of the Odic et al. (2014) task. Children were individually tested in a sound attenuated room. They sat approximately 40 cm away from a 13" Apple Macbook laptop, on which they saw stimuli presented using a custom-made Java program.

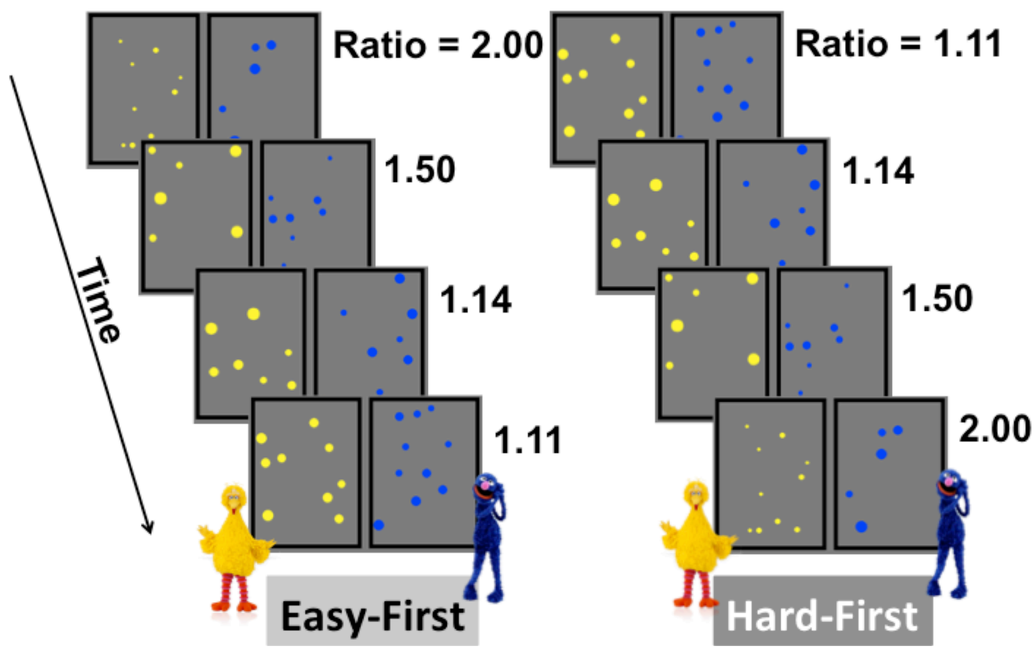


Figure 1. Example stimuli seen by children in the Easy-First and Hard-First training conditions of the ANS Training task in all the Experiments.

Children were told that they were going to play a game in which they would see dots on the screen, and would have to indicate whether more of the dots were yellow or blue. The screen was divided into two sections outlined by two black frames, each with a cartoon figure (Big Bird or Grover) appearing beside it (see Figure 1). On each trial, yellow dots always appeared in the left frame (Big Bird's box), and blue dots in the right frame (Grover's box), and were presented simultaneously. Dots remained visible for 1200 ms, after which they disappeared, leaving only the empty frames and the Big Bird and Grover characters. When children indicated their response either by pointing to or naming the more numerous color, the experimenter immediately pressed the corresponding key on the keyboard ('f' for 'yellow', 'j' for 'blue'), with the computer recording all responses. Children received feedback after every trial: a pre-recorded voice said, "That's right" following correct responses, and "Oh, that's not right" following incorrect ones. The task began with four practice trials of moderate difficulty, during which the two dot arrays first appeared sequentially for an initial exposure, briefly disappeared, and then appeared again simultaneously. The experimenter prompted children to say whether there were more yellow dots or blue dots, and children received feedback as in the test trials. After the four practice trials, the test trials began, in which the arrays were always presented simultaneously.

The numerical ratio between the two dot arrays varied across trials and was either (from hardest to easiest): 1.11 (i.e., 10 dots vs. 9 dots), 1.14 (8 vs. 7), 1.17 (14 vs. 12), 1.25 (10 vs. 8), 1.50 (9 vs. 6) or 2.00 (10 vs. 5). Each ratio was presented five times with different dot sizes and configurations, yielding a total of 30 trials. The side with the larger number of dots was counterbalanced across trials. To discourage children from relying on the cumulative area of the dots, on half of the trials the array with the larger number also had more cumulative area (Congruent trials), and on the other half the array with the larger number had less cumulative area (Incongruent trials).

Children in the Easy-First training Condition were tested in a pre-generated order of trials that began with the easiest ratios (e.g., 2.00 and 1.50) and gradually moved toward the most difficult ones (e.g., 1.14 and 1.11; see Appendix A precise order of trials). For children in the Hard-First training Condition, this trial sequence was exactly reversed. For children in the Random control Condition, trials were intermixed based on their difficulty, but each child saw the same order of trials. All other aspects of the task and displays (e.g., spatial position of individual dots and side of presentation) were identical between the three conditions. The total testing time in this task was approximately 5 minutes.

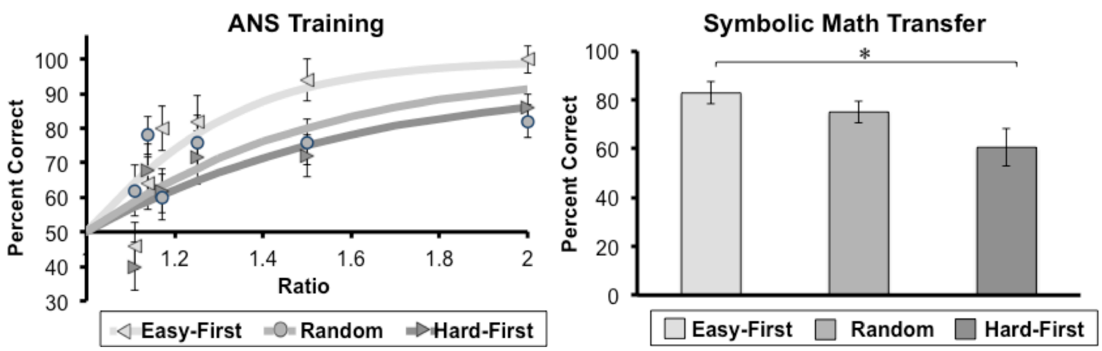
Symbolic Math Transfer task. To measure children's symbolic math ability we administered 18 selected items from Form A of the Test of Early Mathematics Ability (TEMA-3, Ginsburg & Baroody, 2003) immediately following children's completion of the ANS Training task. The TEMA-3 is a standardized test of symbolic math ability that is normed for children between the ages of 3 and 8 years. It contains 72 items, divided into subcategories of informal math abilities (e.g., verbally counting the number of black circles on a page; solving word problems using tokens or fingers) and formal math abilities (e.g., reading and writing Arabic numerals).

We presented only 18 selected items (rather than administering the whole TEMA) for two reasons. First, previous work has found that not all subcategories of mathematical abilities tested by the TEMA-3 correlate with children's ANS precision: while ANS correlates with all of the subcategories in the informal category, it only correlates with the "numeral literacy" in the formal category (Libertus et al., 2013a). Therefore, we selected 18 items from the informal subcategories (numbering, number comparison, and calculation) and the "numerical literacy" subcategory of the TEMA-3 that were within the expected ability range of typically developing 4- to 5-year-old children (See Appendix B for a complete list of test items). We administered all 18 items to children, rather than following the TEMA procedure of stair-casing each child (i.e., continuing until children erred on five consecutive items), and administered them to all children in the same order. As per the TEMA-3 testing manual, children were given only neutral positive feedback throughout this task. The total testing time was approximately 15 minutes. The Symbolic Math Transfer task was always administered after the ANS Training task.

Results

We first analyzed children's performance in the ANS Training task in terms of percent correct across the different numerical ratios. Collapsing across the Easy-First, Hard-First, and Random Conditions, our preliminary analyses revealed no significant differences in children's performance on trials when cumulative area was Congruent versus Incongruent with number ($F(1,18) = .91$, $p = .35$, $\eta_p^2 = .05$), and therefore we collapsed across these two trial types for further analyses.

A 6 (Numerical Ratio) x 3 (Condition: Easy-First, Hard-First, or Random) repeated measures ANOVA with Accuracy on the ANS Training task as the dependent



(a) (b)

Figure 2. (a) Children's performance in the ANS Training task of Experiment 1a. (b) Children's performance in the Symbolic Math Transfer task of Experiment 1a separated by ANS Training Condition – i.e., the ANS trial order that the child had participated in just prior to testing symbolic math (i.e., Easy-First, Hard-First, or Random). Error bars indicate ± 1 standard error.

measure revealed a significant effect of Ratio on children's accuracy ($F(5,135) = 13.91$, $p < .001$, $\eta_p^2 = .34$; all repeated measures analyses in this paper are corrected for non-sphericity), with children performing better on easier ratios, consistent with the psychophysical signature of the Approximate Number System. Critically, we also observed a significant effect of Condition ($F(1,27) = 3.82$, $p = .035$, $\eta_p^2 = .22$), and a marginal interaction between Condition and Ratio ($F(10,135)=1.85$, $p = .08$). Tukey's HSD post-hoc analyses on the effect of Condition showed that children in the Easy-First condition (M = 77.55%, *SD* = 8.09%) performed significantly better than children in the Hard-First condition (M = 66.60%, *SD* = 7.28%; $p < .05$), replicating the ANS confidence hysteresis effect (Odic et al, 2014); children in the Random condition showed intermediate performance (M = 72.33%, *SD* = 11.00%), and were not significantly different from either Easy-First or Hard-First conditions. This suggests that that experiencing confidence hysteresis both enhanced performance (for Easy-First condition) and impaired it (for Hard-First condition).

Using the same psychophysical model and fitting methods used by Odic et al. (2014), we found that the *w* value that resulted in the best fit of the model was 0.20 ($r^2 = .81$) for children's performance in the Easy-First condition, 0.41 ($r^2 = .63$) for children in the Hard-First condition, and 0.33 ($r^2 = .81$) for performance in the Random condition. Because lower values for *w* indicate better precision, this means that children in the Easy-First condition showed better ANS precision than children in the Random condition. In fact, children in the Easy-First condition tended to have better *w* values than previous estimates for the same age range (Halberda & Feigenson, 2008; Piazza et al., 2010; Odic, Libertus, Feigenson & Halberda, 2013), suggesting that experiencing the Easy-First ANS training improved children's ANS precision. In contrast, children in the Hard-First condition showed worse ANS precision than Random control condition. In fact, children in the Hard-First condition had *w* values that were at a level roughly equivalent to that observed for 9-month-old infants (Xu & Spelke, 2003), suggesting that this brief Hard-First ANS training had a negative impact on children's ANS precision. These results replicated the ANS confidence hysteresis effect observed by Odic et al. (2014).

Having replicated the ANS confidence hysteresis effect, we next turned to our central question of whether changing the observed precision of children's ANS representations also changes their performance in a Symbolic Math Transfer task. A one-way ANOVA with Symbolic Math Transfer accuracy as a depedennt variable showed a significant main effect of Condition ($F(2,27) = 3.89$, $p = .033$, η_p^2=.22: children in the Easy-First ANS training Condition performed better in the Symbolic Math Transfer task

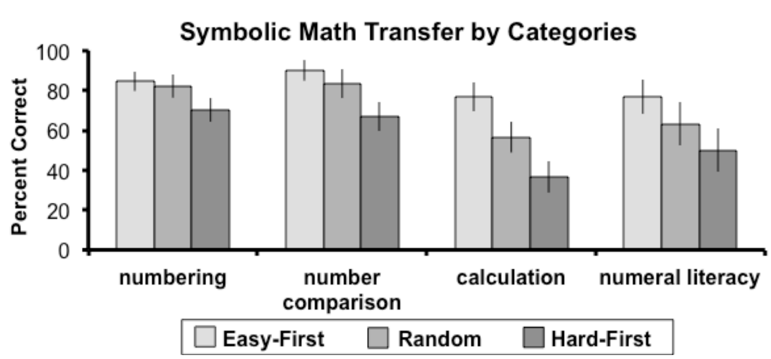


Figure 3. Five year old's performance in different Categories of the Symbolic Math Task, separated by ANS Condition. The bars indicate SEM.

(82.78%, *SD* = 14.21%) than children in the Hard-First ANS training Condition (60.56%, *SD* = 24.21%), and children in the Random control condition performed at intermediate levels (75%, *SD* = 13.92%). Consistent with this, Tukey's HSD post-hoc analyses revealed a significant difference between Easy-First and Hard-First conditions ($p < .05$), but no significant difference between either Easy-First and Random, or Hard-First and Random. These results suggest that children's Symbolic Math Transfer task performance was boosted by the Easy-First ANS training and negatively affected by the Hard-First ANS training.

Our Symbolic Math Transfer task included 4 different Categories (Numbering, Number Comparison, Calculation, and Numeral Literacy) from TEMA, allowing us to test whether ANS precision is uniquely related to specific math abilities captured by these items. To ask whether the transfer effect of Easy-First and Hard-First ANS training was present in all 4 Categories of the Symbolic Math Transfer task, data from all items in each Category were averaged to calculate individual percent correct in each Category of the Symbolic Math Transfer task. We found a main effect of Category ($F(3, 81) = 8.97$, $p = .001$, η_p^2=.25): average performance across all children for Numbering was 78.89% (*SD* = 19.2%), for Number Comparison was 80.00% (*SD* = 24.13%), for Calculation was 56.67% (*SD* = 29.23%), and for Numeral Literacy was 63.33% (*SD* = 35.40%), suggesting that Numeral Literacy and Calculation abilities emerge later for preschoolers than Numbering and Number Comparison (consistent with Ginsburg & Baroody, 2003). But, critically, we found no Condition x Category interaction ($F(6, 81) = 0.69$, $p = .58$, η_p^2=.05), suggesting that 5-year-old children's performance in different types of symbolic math items was equally affected by manipulations of ANS precision.

Discussion

The results from Experiment 1a replicated the effect of confidence hysteresis in preschool children's performance when making non-symbolic approximate numerical judgments (Odic et al., 2014): children's ANS precision was significantly better in the Easy-First training condition than the Hard-First training condition even though they saw exactly the same trials – just in a reversed order. Critically, we also found that this temporary change in ANS precision transferred to children's performance on a subsequent Symbolic Math Transfer task: children in the Easy-First condition performed better in the Symbolic Math Transfer task than children in the Hard-First condition. Additionally, results from the Random control condition showed that this difference was driven by both a boost in performance in the Easy-First ANS training group and impairment in performance in the Hard-First ANS training group.

Our results are consistent with at least two interpretations. First, there may be a direct causal link between ANS precision and symbolic mathematics and, hence, modifying ANS precision through confidence hysteresis may have specifically changed children's symbolic math performance. Alternatively, the confidence hysteresis effect could have broadly influenced children's motivation or self-efficacy, such that children in the Easy-First training condition felt more motivated and confident than children in the Hard-First training condition. Although previous work has suggested that the decrease in performance during the Hard-First condition itself is not a byproduct of a general decrease in motivation (Odic et al., 2014), it is still possible that once the ANS training session had ended, children who just finished the Hard-First condition may have felt demotivated to perform any subsequent tasks.

To test whether the observed transfer effect of ANS confidence hysteresis was specific to the number domain or instead was due to general motivational changes, we conducted a second experiment in which we tested the effect of confidence hysteresis on a subsequent verbal task. Verbal tests provide three characteristics that make them an appropriate control for the effect observed in Experiment 1a. First, there are no known correlations between ANS precision and verbal knowledge measures, and, hence, if the confidence hysteresis effect is specific to the ANS, there should be no change in verbal ability as a result of training condition (for this reason, verbal knowledge was also used as a control task in Park & Brannon, 2013, 2014, and Hyde et al., 2014). Second, social cognitive factors such as motivation and anxiety have been shown to alter performance in not only math (Beilock, 2008; Dweck, 1986), but also verbal tasks such as verbal recall (Gray, 2001), verbal labeling (Dusek, Kermis & Mergler, 1975) and the acquisition of new words (Maclntyre & Gardner, 1994), as well as general memory or task-solving abilities (Eysenck & Calvo, 2008). Hence, if confidence hysteresis affects general motivation, we should also find a group difference in performance on the vocabulary task.

Finally, there are numerous standardized vocabulary tests that, much like the TEMA-3, have been normed for preschoolers (e.g., the Peabody Picture Vocabulary Test; Dunn & Dunn, 2007). This makes it possible to administer a vocabulary task that is of roughly equal difficulty to our Symbolic Math Transfer task, thus eliminating the possibility of not observing an effect of transfer due to floor or ceiling performance.

In Experiment 1b, a separate group of children first completed the same ANS Training task as in Experiment 1a, with half of the children tested in the Easy-First training condition and half in the Hard-First training condition. Children were then tested in a Vocabulary Transfer task that assessed their ability to identify which of several pictures corresponded to a spoken word. If the transfer effect we observed in Experiment 1a was caused by the changes in children's general confidence or motivation, then children in the Easy-First training condition of the ANS Training task should perform better in the Vocabulary Transfer task than children in the Hard-First training condition. In contrast, if confidence hysteresis selectively affected children's numerical abilities, then children's vocabulary performance should not differ between the two training conditions.

Experiment 1b: ANS Confidence Hysteresis and Vocabulary in 5-Year-Olds

Method

Participants. Twenty children with a mean age of 5 years, 3 months (*SD* = 2.15 months; range = 59.11 months to 66.35 months; 9 females) participated in Experiment 1b, with 10 children assigned to each training Condition (Easy-First or Hard-First). All children then completed the Vocabulary Transfer task. None of the children participated in Experiment 1a. Three additional children (all assigned to the Hard-First ANS training condition) were tested but excluded from the final sample because they failed to correctly answer more than one of the first three questions (i.e., the easiest three questions) of the Vocabulary Transfer task, suggesting they did not understand the task. Preliminary analysis showed no significant difference in the average age of children in the two Conditions across the two Experiments ($F(1, 36) = 2.5$, p=.12, η_p^2= .07), and there was no Condition x Experiment interaction ($F(1,36) = 1.72$, p=.20, η_p^2= .05).

ANS Training task. This was identical to that in Experiment 1a, with children assigned to either the Easy-First or the Hard-First training Condition. The same numerical ratios, display time, and feedback were used.

Vocabulary Transfer task. To measure children's vocabulary, we administered 24 selected items from the Peabody Picture Vocabulary Test (PPVT-4; Dunn & Dunn, 2007). The PPVT-4 consists of a picture book with four colored drawings per page. For each page the experimenter read a word aloud (e.g., "farm"; "catching"; "digital") and children point to the picture that best matches the meaning of the word. The PPVT-4 is normed for English speakers between the ages of 2 years and 6 months through adulthood (81+). To make the task comparable to the Symbolic Math Transfer task from Experiment 1, we selected 24 items that were predicted to fall within the full range of vocabulary knowledge of 5-year-old children. As in the Symbolic Math Transfer task from Experiment 1a, we presented all vocabulary test items in the same sequence for all children, and provided no feedback on their performance. As in Experiment 1a, the ANS Training task was always administered first.

Results

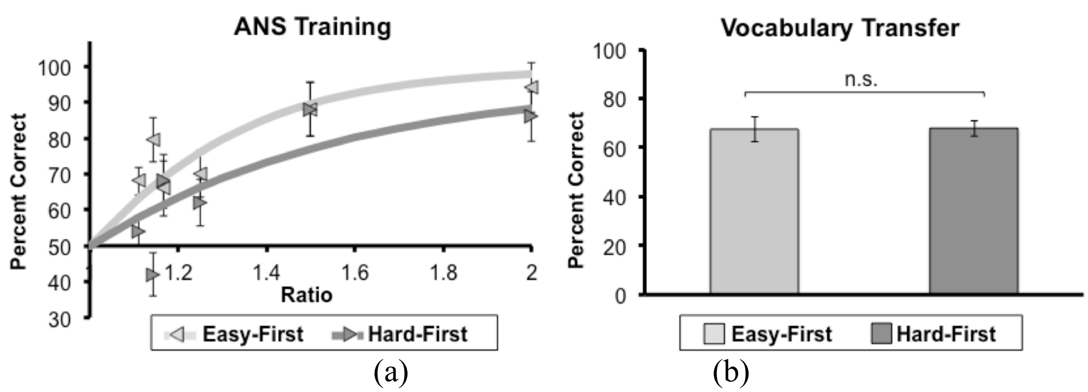


Figure 3. (a) Children's performance in the ANS Training task of Experiment 1b. (b) Children's performance in the Vocabulary Transfer task of Experiment 1b separated by ANS Training Condition (i.e., Easy-First or Hard-First). Error bars indicate ± 1 standard error.

The confidence hysteresis effect replicated in Experiment 1b: a 6 (Numerical Ratio) x 2 (Training Condition: Easy-First or Hard-First) repeated measures ANOVA with Accuracy on the ANS Training task as the dependent measure showed an effect of Condition ($F(1,18) = 4.75$, $p = .04$, $\eta_p^2 = .21$), a significant effect of Ratio ($F(5,90) = 9.38$,

$p < .001$, $\eta_p^2 = .34$), and a marginal interaction between Ratio and Condition ($F(5,90)=2.38$, $p = .08$, $\eta_p^2 = .12$). As shown in Figure 3, children in Easy-First Condition responded correctly on 76.13% of the trials ($SD = 8.31\%$; $w = 0.22$, $r^2 = .74$), which was significantly better than Hard-First children's performance (67.00%, $SD = 10.71\%$; $w = 0.37$, $r^2 = .73$; $F(1,18) = 4.75$, $p = .04$, $\eta_p^2 = .21$). Further analyses showed that there was no significant difference between accuracy in the ANS Training task across experiments ($F(1,36) = .03$, $p = .85$), and no Experiment x Condition interaction ($F(1,36) = .11$, $p = .74$).

If the results in Experiment 1a are due to confidence hysteresis affecting general motivation, we should find transfer to the Vocabulary Transfer Task. Contrary to this, there was no significant effect of ANS training Condition on children's subsequent vocabulary performance ($F(1,18) < .01$, $p = .95$): children in the Easy-First Condition performed 67.50% correct on the Vocabulary Transfer task ($SD = 16.17\%$), and children in the Hard-First Condition performed 67.92% correct ($SD = 10.22\%$). Furthermore, there was a significant Experiment x Condition interaction ($F(1,36) = 4.44$, $p = .04$, $\eta_p^2=.11$) and no main effect of Experiment ($F(1,36) = .54$, $p = .47$), suggesting that we were successful in matching the difficulty between the Symbolic Math Transfer task and the Vocabulary Transfer task. ANS confidence hysteresis thus only affected subsequent symbolic math performance (Experiment 1a), and not subsequent task of vocabulary performance (Experiment 1b). These results suggest that the ANS confidence hysteresis effect did not impair children's domain-general motivation or self-efficacy, and that temporary modulations of ANS precision have a direct and unique effect on symbolic math performance.

Experiment 2a: ANS Confidence Hysteresis and Symbolic Math in 4-Year-Olds

Thus far, our results suggest a direct causal link between ANS precision and formal mathematics in 5-year-old children: temporary modulations of ANS precision due to confidence hysteresis transfer to a symbolic math task, but not to an equally difficult vocabulary task.

Next, we ask whether this effect is present even earlier in development. Testing the transfer effect in younger children allows to both replicate our findings and to investigate a novel question of whether there are any developmental changes in the relationship between ANS precision and symbolic math. The gradual emerging relationship between the ANS and symbolic math is especially interesting in 4-year-olds, as most of these children have just learned how to count (Wynn, 1992; Le Corre & Carey, 2007; Sarnecka & Carey, 2008). The link between ANS and symbolic math may also be built step-by-step while children acquire different levels of symbolic math skills, which would predict that ANS training influences younger children's performance on only the subset of symbolic math skills that are linked with ANS early on.

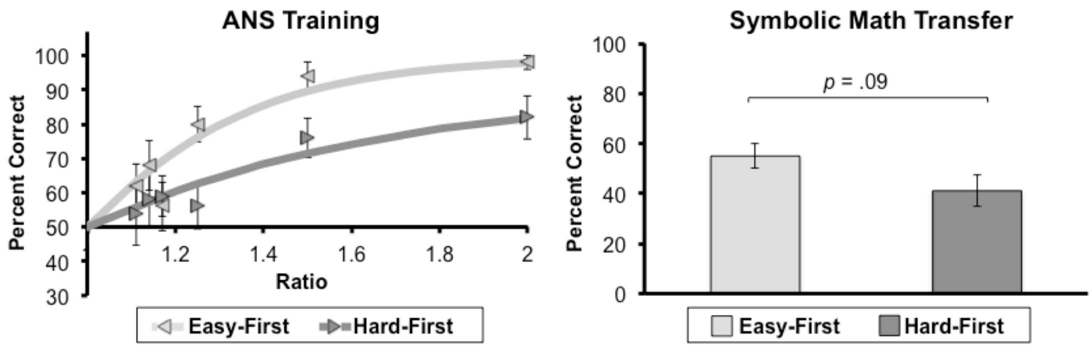
In Experiment 2a, we gave the same ANS Training task to a group of 4-year-old children, with half of them assigned to the Easy-First ANS training condition, and half to the Hard-First ANS training condition. Subsequently, they were tested in the same Symbolic Math Transfer task as in Experiment 1a. If the causal link between ANS and symbolic math is present as soon as children understand number symbols, we should observe a similar transfer effect of ANS confidence hysteresis on 4-year-olds' symbolic math performance. In other words, children in the Easy-First ANS training group should perform better in the subsequent symbolic math task than children in the Hard-First group. Results from the following experiment would address the question of how and when is children's symbolic math performance subjective to changes in the ANS.

Method

Participants. Twenty children with a mean age of 4 years, 8 months (*SD* = 1.45 months; range = 55.20 months to 59.70 months; 10 females) participated in Experiment 2a, with 10 children assigned to each training Condition (Easy-First or Hard-First). All children then completed the Symbolic Math Transfer task. Three additional children (1 of them were assigned to the Hard-First ANS training condition) were tested but excluded from the final sample because they failed to correctly answer more than one of the first three questions (i.e., the easiest three questions) of the Symbolic Math Transfer task.

ANS Training task. This was identical to that in Experiments 1a and 1b, with children assigned to either the Easy-First or the Hard-First training condition. The same numerical ratios, display time, and feedback were used.

Symbolic Math Transfer task. This was also identical to that in Experiment 1a. The same test items and trial order were used.



(a) (b)

Figure 4. (a) Children's performance in the ANS Training task of Experiment 2a. (b) Children's performance in the Symbolic Math Transfer task of Experiment 2a, separated by ANS training Condition (Easy-First or Hard-First). Error bars indicate ± 1 standard error.

Results

As in Experiment 1a, preliminary analyses found no effects of cumulative area (Congruent vs. Incongruent) on children's numerical discriminations ($F(1,18) = .01$, $p = .93$, $\eta_p^2 < .01$), and therefore we collapsed across these two trial types for further analyses. We once again replicated the effect of ANS confidence hysteresis: a 6 (numerical Ratio) x 2 (ANS Training Condition: Easy-First or Hard-First) repeated measures ANOVA with Accuracy on the ANS Training task as the dependent measure revealed an effect of ANS Training Condition ($F(1,18) = 8.35$, $p = .01$, $\eta_p^2 = .32$) and ratio ($F(5,90) = 9.85$, $p < .001$, $\eta_p^2 = .35$) and no interaction between Ratio and Condition ($F(5,90)=1.11$, $p = .36$, $\eta_p^2 = .06$). As shown in Figure 5, children in Easy-First Condition responded correctly on 76.33% of the trials (SD = 5.54%; $w = 0.22$, $r^2 = .88$) and children in the Hard-First Condition responded correctly on 64.16% (SD = 12.11%, $w = 0.49$; $r^2 = .92$).

There was a marginal effect of ANS training Condition on subsequent Symbolic Math Transfer accuracy ($F(1,18) = 3.14$, $p = .09$, $\eta_p^2 = .15$): children in the Easy-First training condition performed 55.00% correct on the Symbolic Math Transfer task (SD = 15.15%), and children in the Hard-First training condition performed 41.11% correct (SD

= 19.63%). Hence, there is a trend for children in the Easy-First ANS training Condition to perform better in the Symbolic Math Transfer task than children in the Hard-First ANS training Condition, consistent with Experiment 1a.

One possible reason for this marginal effect may be that ANS confidence hysteresis only has an effect on a subset of items in 4-year-old children (perhaps because not all math abilities have formed links to the ANS). Indeed, we found that 4-year-olds, as a group, performed significantly more poorly in the Symbolic Math Transfer task than the 5-year-olds in Experiment 1a ($F(1,36) = 15.90$, $p <.001$, $\eta_p^2 = .31$). Among the four Symbolic Math Transfer task Categories, Numbering (knowledge about the counting list) and Number Comparison (comparing numbers verbally) are known to be mastered earlier in development than Calculation (solving verbal arithmetic problems) and Numeral Literacy (reading and writing numerals; Ginsburg & Baroody, 2003; Denton & West, 2002). This pattern was replicated in Experiment 1a amongst five-year-olds; similarly, 4-year-olds performed better on the Numbering (M = 60.56%, *SD* = 21.77%) and Number Comparison Categories (M = 48.33%, *SD* =39.70) compared to the Calculation (M = 26.67%, *SD* = 25.60%) and Number Literacy (M = 31.67%, *SD* = 31.48) Categories.

Consistent with a developmental modulation of the ANS transfer effect, we found a significant main effect of Category ($F(3, 54) = 7.05$, $p = .001$, $\eta_p^2 = .28$) and a significant ANS Training Condition x Category interaction ($F(3, 54) = 3.85$, $p = .024$, $\eta_p^2 = .18$) on children's performance. Post hoc contrasts showed that the Condition x Category interaction was driven by the difference between Easy-First and Hard-First groups' performance in Numbering combined with Number Comparison versus Calculation combined with Numeral Literacy ($F(1,54) = 13.24$, $p < .05$, accounted for 80% of the interaction, corrected by Sheffe's for post hoc comparisons), suggesting that ANS training was effective on 4-year-olds' performance in the symbolic math items that belong to Numbering and Number Comparison Categories, but not those belong to Calculation and Numeral Literacy. Furthermore, when combined with the results from Experiment 1a, we found a significant Age x Condition x Category interaction ($F(3, 108) = 4.22$, $p = .015$, $\eta_p^2 = .11$), suggesting the effect of ANS Training was different between two age groups across different math Categories.

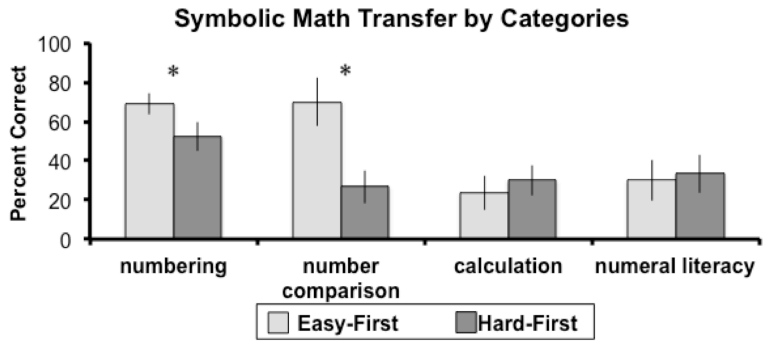


Figure 6. Four-year-old children's performance in different Categories of the Symbolic Math Transfer task separated by ANS training Conditions. Error bars indicate ± 1 standard error.

Importantly, the interaction between the ANS training, Age and Symbolic Math Categories cannot be accounted by a simple floor effect, as "chance" performance for these symbolic math questions would be ~0% correct, whereas both Easy-First and Hard-First groups were about 30% correct for these categories (Easy-First: 23% for calculation and 30% for numeral literacy; Hard-First: 30% and 33%), which was significantly above 0% ($t(19)$s > 2.5, *ps* < .05). In addition, it cannot be accounted simply by difficulty of the math task, as splitting the items by difficulty (based from the TEMA-3 norms; Ginsburg & Baroody, 2003) revealed no effect of Difficulty ($F(1,18) = 0.2$, $p = .66$). Instead, our results suggest that ANS training selectively influences children's performance based on the required symbolic math skills. Thus the link between precision in the ANS and symbolic math performance might be built step-by-step during acquisition of different types of symbolic math skills.

Experiment 2b: ANS Confidence Hysteresis and Phonological Processing in 4-Year-Olds

Thus far, results from Experiments 1a, 1b, and 2a have shown a direct causal effect of ANS confidence hysteresis on preschool children's symbolic math performance. The absence of the ANS confidence hysteresis effect on children's vocabulary performance in Experiment 1b suggests that the effect was not due to changes in children's motivation to perform any subsequent task.

Experiment 2b had two goals: first, to replicate the finding that ANS confidence hysteresis does not transfer to subsequent verbal tests, and, second, to test whether a verbal task that requires more online and working-memory processing would be affected by ANS confidence hysteresis. The latter issue is especially important since the vocabulary task administered in Experiment 1b required children to interpret pictures and retrieve information from long-term memory. This leaves open the possibility that a non-numerical task that requires children to keep information in working memory and to process information online would be influenced by ANS confidence hysteresis. For example, phonological processing is the ability to identify speech sounds and combine sounds together to form words. Moreover, it has been suggested that phonological awareness plays a role in children's symbolic math performance as it mediates the interpretation of math symbols and the retrieval of math facts (De Smedt, Taylor, Archibald & Ansari, 2009). If ANS confidence hysteresis influences children's performance on the Symbolic Math Transfer task due to changes in general motivation, we would expect to find that children who experienced the Hard-First ANS training perform worse in a subsequent phonological task compared to those in the Easy-First ANS training condition. However, if ANS confidence hysteresis influences symbolic math performance via a link between making decisions based on the ANS and using symbolic math skills, we would find no difference between the two groups on the phonological task.

Method

Participants. Twenty children with a mean age of 4 years, 9 months (*SD* = 1.81 months; range = 54.80 months to 59.97 months; 9 females) participated in Experiment 1b, with 10 children assigned to each training Condition (Easy-First or Hard-First). All children then completed the Phonological Transfer task. Twelve additional children were tested but excluded from the final sample because of parental interference (one child), refused to continue with the experiment (two children, both in Hard-First), or failed to correctly answer more than one of the first three questions (i.e., the easiest three questions) of the Phonological Transfer task (4 in Easy-First, 5 in Hard-First). Preliminary analysis showed no significant difference in the average age of children in the two Conditions across the two Experiments ($F(1, 36) = .22$, p=.64), and there was no Condition x Experiment interaction ($F(1,36) = .30$, p=.59).

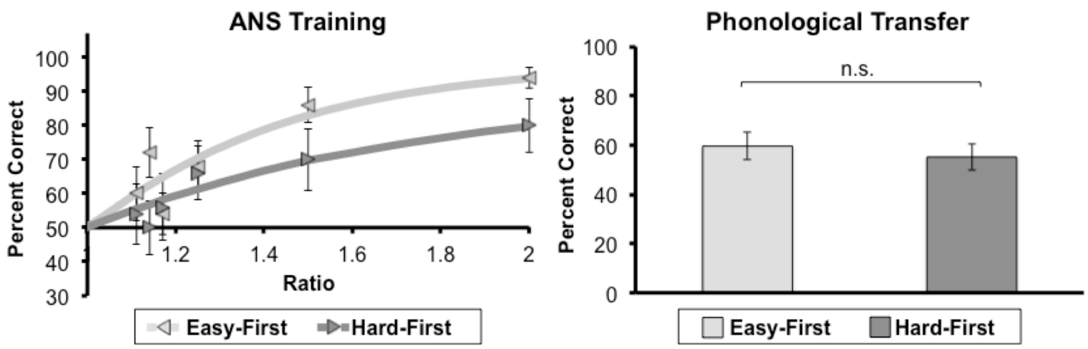
ANS Training task. This was identical to that in Experiment 1a, with children assigned to either the Easy-First or the Hard-First training Condition. The same numerical ratios, display time, and feedback were used.

Phonological Transfer task. To measure children's phonological awareness, we administered 18 selected items from the blending words subtest of the Comprehensive Test of Phonological Processing (CTOPP; Wagner, Torgesen & Rashotte, 1999). The CTOPP is comprised of three subtests (rapid naming, phonological memory and phonological awareness) that are normed for participants ranging from 5 years to 24 years. This test measures individuals' ability to combine speech sounds to form words. For example, children would hear recordings of a series of syllables separated by short temporal gaps, such as "can-dy", and be asked to put the sounds together to make a word, in this case the correct answer is "candy". As in the Symbolic Math Transfer task from Experiment 2a, we presented all phonological test items in the same sequence for all children, and provided no feedback on their performance. As in Experiment 1a, the ANS Training task was always administered first.

Results

As in previous experiments, preliminary analyses found no effects of cumulative area (Congruent vs. Incongruent) on children's numerical discriminations ($F(1,18) = 1.08$, $p = .31$, $\eta_p^2 = .06$), and therefore we collapsed across these two trial types for further analyses. We again replicated the effect of confidence hysteresis: there was a significant effect of Condition ($F(1,18) = 5.93$, $p = .026$, $\eta_p^2 = .25$) and Ratio ($F(5,90) = 5.35$, $p = .002$, $\eta_p^2 = .23$) on children's performance, and there was no interaction between Ratio and Condition ($F(5,90)=.70$, $p = .57$). As shown in Figure 6a, children in Easy-First Condition responded correctly on 72.33% of the trials (SD = 8.47%; $w = 0.29$, $r^2 = .82$) , and children in the Hard-First Condition responded correctly on 62.67% (SD = 9.27%; $w = 0.54$, $r^2 = .91$). Further analyses showed that there was no significant difference between accuracy in the ANS Training task Experiments 2a and 2b ($F(1,36) = .90$, $p = .35$), and there was no Experiment x Condition interaction ($F(1,36) = .19$, $p = .67$).

Lastly, we examined children's performance in the Phonological Transfer task. Children in the Easy-First Condition performed 59.56% correct on the Vocabulary Transfer task (*SD* = 18.24%), and children in the Hard-First Condition performed 55.17% correct (*SD* = 15.62%). Hence, children in different training conditions of the ANS Training task did not differ in their performance in the Phonological Transfer task ($F(1,18) = .33$, $p = .57$). Additionally, we found no main effect of Experiment across Experiments 2a and 2b ($F(1,36) = 2.91$, $p = .10$), suggesting that we were successful in



(a) (b)

Figure 7. (a) Children's performance in the ANS Training task of Experiment 2b. (b) Children's performance in the Phonological Transfer task of Experiment 2b, separated by ANS training Condition (Easy-First or Hard-First). Error bars indicate ± 1 standard error.

matching the difficulty between the Symbolic Math Transfer task and the Phonological Transfer task for 4-year-old children.

Results from Experiment 2b showed that ANS confidence hysteresis did not affect performance in a subsequent phonological task, which is inconsistent with the general motivation explanation for the observed transfer on symbolic math performance. Instead, findings from all four experiments suggest that changes in the ANS due to confidence hysteresis influences symbolic math performance in a domain-specific way that is most likely due to the involvement of ANS-based decision-making processes when children solve symbolic math problems.

General Discussion

There have been numerous recent reports showing a correlation between the intuitive ANS and symbolic mathematics abilities; but, to date, no study has shown a direct causal link between ANS *precision* and symbolic mathematics. Here, we investigated whether ANS precision contributes causally to individual differences in symbolic mathematics prior to any formal mathematics education. Our experiments revealed three main findings. First, all of our experiments replicated the effect of ANS confidence hysteresis, in which the trial order in which participants perform ANS discriminations significantly affects their observed ANS precision (Odic et al., 2014). Children who completed a series of ANS discriminations that progressed from easy ratios to hard ones showed significantly better ANS precision than children who completed the same discriminations in the reverse order. This result expands on the emerging body of research on the malleability of ANS precision, suggesting that accuracy in making approximate numerical judgments is sensitive to experimental manipulations such as computer-assisted intervention (Wilson et al., 2006; Räsänen et al., 2009), the order of trial difficulty (Odic et al., 2014) and the presence of feedback (DeWind & Brannon, 2012; but see Lindskog, Winman & Juslin, 2013 for alternative interpretation).

Our most important finding is that the temporary change in non-symbolic numerical discrimination transferred to children's performance on a Symbolic Math Transfer task. We found that 4- and 5-year-old children performed significantly better on a subsequent test of symbolic math ability after having performed a series of ANS judgments in a Easy-First trial order, compared to children who performed the same task in a Hard-First trial order. This is the first demonstration of a causal link between ANS precision and symbolic math performance in which children had received little or no formal math education (e.g., unlike Hyde et al., 2014). The observed effect of ANS confidence hysteresis on symbolic math performance was not restricted to the tasks that share the non-symbolic aspect of the ANS Training task, such as magnitude comparison (e.g., as in Räsänen et al., 2009) or basic arithmetic (e.g., as in Park & Brannon, 2013, 2014). Rather, we demonstrated that changes in 5-year-olds' ANS precision affect a broad range of symbolic math abilities, including solving word problems, counting, and reading numerals. These findings support the view that practice performing non-symbolic arithmetic computations (approximate addition and subtraction) is not necessary to achieve an improvement in symbolic math performance (see Hyde et al., 2014 and Park & Brannon, 2013). Impressively, this transfer from ANS confidence hysteresis to symbolic math performance was observed after children had completed only about 5 minutes worth of ANS training. Thus, the present results demonstrate a rapid change in observed ANS precision from a brief intervention.

Third, we found that 4-year-olds, in contrast to 5-year-olds, only showed transfer for a subset of Symbolic Math Transfer task items. Further analyses showed that the transfer effect was not modulated by the difficulty of the math task, but rather by the specific math categories (and, hence, math ability) tested. Specifically, more elementary math skills such as counting and comparing numbers were affected by ANS confidence hysteresis in 4-year-olds, while more advanced math skills such as reading numerals and solving word problems were not. These results suggest the link between ANS and symbolic math is built as children are acquiring specific symbolic math abilities. This would predict that for adults who have been well equipped with all kinds of symbolic math skills, ANS confidence hysteresis would influence their performance in a wider range of symbolic math tasks. As we elaborate below, this may provide important clues about the mechanism that links the ANS with symbolic math abilities. Future work should explore whether individual differences in ANS precision accelerate the acquisition of some of these math abilities.

Fourth, we found that the effects of ANS confidence hysteresis did not transfer to performance on subsequent non-numerical tasks (a vocabulary task or a phonological task), in spite of the tasks being matched in overall difficulty. This suggests that the changes we observed in children's symbolic math performance were unlikely to be due to confidence hysteresis affecting motivation or general self-confidence. In fact, if changes to motivation or self-confidence were responsible for the transfer effect, we might expect an opposite pattern to that which we observed: children assigned to the Easy-First training condition experienced the hardest trials at the very end of the training task, whereas children in the Hard-First training condition experienced the easiest ANS trials right before the transfer tasks. Thus, if the experiences of the children just prior to the transfer task that were critical in driving subsequent performance, then the results on the transfer task would have been exactly the opposite of what we observed. Hence, the current results suggest that manipulation of ANS precision alters symbolic math performance in a way that is relatively independent of the social-cognitive factors that have been previously identified to play a role in altering math performance, such as motivational goals (Dweck, 1986) and self-efficacy beliefs (Hackett & Betz, 1989), though we strongly believe that these factors are critical for children's math performance, as well.

Why does manipulating children's ANS precision alter their symbolic math performance? One possibility is that symbolic numbers are represented directly by the ANS, perhaps through a quick re-tuning of the neurons coding for ANS representations (Piazza, 2010), such that every time a symbolic number representation is activated, a corresponding internal signal from the ANS is involved. Thus, when precision of the ANS representations is improved through the scaffolding sequence of numerical discriminations in the Easy-First training condition, symbolic number representations that activate the ANS representations become more reliable to children as well. In contrast, when the reliability of the ANS signals is degraded due to the Hard-First ANS experience, symbolic math performance is diminished. Under this view, symbolic numbers are represented directly through the ANS (Dehaene, 1997; Piazza, Pinel, Le Bihan & Dehaene, 2007), and the ANS can act as a kind of precise mental calculator. One challenge for this theory, however, is that the continuous, relative nature of the ANS makes it poor for solving exact math problems: even in ideal situations, the ANS is not the kind of tool that can effectively act as a precise mental calculator (Halberda & Odic, in press).

An alternative explanation is that symbolic math computations occur outside the ANS, but that the ANS plays a role in supporting children's interpretation about math problems and provides them them ball-park estimations for the correct answers, thus helping children to solve symbolic math problems more accurately and efficiently (Jordan, Glutting, & Ramineni, 2010; Greeno, 1991). For example, when encountering a math problem like "14+3", children who have been scaffolded to have higher confidence in their ANS representations may spontaneously make use of ANS representations to approximate the quantities corresponding to the sum of 14 and 3, and use these intuitions to support finding the right answer with their symbolic math representations. In contrast, children who have been discouraged from using their ANS representations via the Hard-First training condition may rely solely on procedural knowledge to solve such math problems, and hence miss out on an important source of evidence and guidance that is active for their peers in the Easy-First training condition.

A third possibility is that ANS precision may play no direct or supporting role as children attempt to solve math problems, but rather that children experience a kind of self-efficacy cost that is specific to the domain of mathematics. That is, ANS confidence hysteresis might affect children's internal sense of how capable they are at thinking about numbers. For example, children who experienced the Easy-First ANS training might have been encouraged to think, "I understood the number guessing game with blue and yellow dots, so I must know this other number game as well". In contrast, children who experienced the Hard-First ANS training might have felt low confidence in making numerical judgments (both approximate, non-symbolic ones and precise, symbolic ones), and thus have performed worse on the subsequent symbolic math test. If this is the case, the selectivity of the transfer effect of ANS confidence hysteresis on 4-year-old children's symbolic math performance suggests that this self-efficacy cost is highly specific to the symbolic math skills that have been linked to ANS representations.

The exact nature of the interactions among these possible links between ANS precision and symbolic math performance remain to be explored. Our discovery of a very brief intervention, confidence hysteresis, that can directly change observed ANS precision for the better (Easy-First training) or for the worse (Hard-First training) may be an important tool for probing these connections via experimental manipulations of ANS precision.

Finally, an interesting question for future research is the potential use of ANS confidence hysteresis as an intervention method for improving children's math abilities. The feasibility of such an intervention depends greatly on the duration of the confidence hysteresis effect. Findings from studies of perceptual hysteresis and the effect of feedback on test performance suggest two alternative possibilities. Previous work on perceptual hysteresis in non-numerical representations (such as apparent motion, Hock & Schoner, 2010) suggests that the confidence hysteresis effect we observed may be due to the observer's experience of only the most recently completed trials. On this account, the effect of hysteresis on either numerical discrimination or on symbolic math performance would not be expected to last longer than a single experimental session. However, findings from studies of ANS precision training in adults suggest that after practicing with trial-by-trial feedback, improvements in adults' performance in a numerical discrimination task can be maintained even when the feedback is removed hours later (DeWind & Brannon, 2012). On this account, effects of ANS confidence hysteresis, and perhaps the transfer to symbolic math performance, might be more enduring. Similarly, the benefits of extended ANS training remain to be explored. As has been demonstrated in the literature on pedagogy, consistent scaffolding in math instruction can lead to enhancement in learning on the long run (Anghileri, 2006). It will be exciting to determine whether extended ANS training has similar benefits.

In summary, the primary question motivating the present research was whether precision of the ANS contributes causally to individual differences in children's mathematical performance, even in children with no school-based formal mathematics education, and when training is non-arithmetic in nature. Here we demonstrated that a brief change in 4- and 5-year-old children's nonverbal numerical discrimination performance, induced by confidence hysteresis, transferred to children's performance on symbolic math tasks but not to their performance on non-numerical verbal tasks. These findings provide direct evidence that the ANS serves as one of the foundations for symbolic mathematics, starting early in development.

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Appendix A.

TrialNum	Yellow	Blue
1	10	5
2	6	9
3	9	6
4	10	5
5	5	10
6	6	9
7	10	8
8	10	5
9	9	6
10	5	10
11	10	8
12	6	9
13	10	8
14	12	14
15	10	8
16	7	8
17	14	12
18	7	8
19	8	10
20	12	14
21	8	7
22	7	8
23	9	10
24	12	14
25	9	10
26	10	9
27	14	12
28	10	9
29	8	7
30	10	9

Appendix B.

Item number in the original test	Stimulus
10	Give me _ tokens. (T)
11	Hold up _ fingers.
13	What number comes next; _, and then comes?
15	Write the number.
16	How many does he have altogether? (T)
19	Which is more, …?
20	Which is more, …?
21	Count up as high as you can. (stop at 21 or 42)
22	What number comes next; _, and then comes?
23	Count these dots with your finger.
24	Count backwards, start from 10.
26	How much are _ and _ altogether? (T)
27	Which is closer to _, _ or _?
29	What number is this?
32	How much is _ and _ altogether? (Must count up from larger added)
35	What number is this?
38	Count these dots with your finger.

Footnotes

[1]One additional child was excluded from the final sample in Experiment 2b due to extremely low performance in the ANS training task compared to the rest of the group.

Figure captions

Figure 1. Example stimuli seen by children in the Scaffolded Easy-First and Challenging Hard-First training conditions of the ANS Training task in Experiment 1 and 2.

Figure 2. (a) Children's performance in the ANS Training task of Experiment 1. (b) Children's performance in the Symbolic Math Transfer task of Experiment 1 separated by ANS Training Condition – i.e., the ANS trial order that the child had participated in just prior to testing symbolic math (i.e., Scaffolded Easy-First or Challenging Hard-First). Error bars indicate ± 1 standard error.

Figure 3. (a) Children's performance in the ANS Training task of Experiment 2. (b) Children's performance in the Vocabulary Transfer task of Experiment 2, separated by ANS training condition – i.e., the ANS trial order that the child had participated in just prior to testing symbolic math (i.e., Scaffolded Easy-First or Challenging Hard-First). Error bars indicate ± 1 standard error.